UNDERGRADUATE MATHEMATICS CLUBS.

EDITED BY R. C. ARCHIBALD, Brown University, Providence, R. I.

The editor earnestly desires that readers should make suggestions to him whereby this department may better serve those for whom it is designed. Very general coöperation in this respect would surely be in the best interests of all concerned.

Every club which has not sent in a complete report of its meetings during 1917–18 is requested to do so as soon as possible.

CLUB ACTIVITIES.

THE MATHEMATICS CLUB OF CONNECTICUT COLLEGE, New London, Conn.

This newest club at the newest New England college was organized last December through the inspiration of Professor D. D. Leib. Membership is open to those who are taking, or have already taken, a course in mathematics beyond the regular freshman requirement. "The two-fold purpose announced by the club was to foster in the college a proper appreciation of the claims of mathematics upon students, and to acquaint its members, by the presentation of formal papers and by informal discussion, with topics of mathematical interest or value along lines not likely to be included in regular class work. While at least one paper on a definite topic has been presented at each meeting, no attempt has been made to confine the informal discussion to any one topic, and the concluding half hour or more is enlivened by simple 'eats' of some sort." The club had seven members this year.

Officers: President, Ruth F. Avery '19; secretary, Justine McGowan '20; treasurer, Dorothy Peck '19. These officers constitute the committee on program and arrangements. The absence of seniors is due to the fact that the college is just in the third year of its history and the first class will be graduated in 1919. The programs of the meetings have been as follows:

December: "Mathematics and Mathematics Clubs at other Colleges" (particularly Women's Colleges)—a general discussion; articles from the MONTHLY and other sources were summarized.

January: "The Fourth Dimension" by Professor Leib.

February: "Magic Squares" by Margaret Maher '19.

March: "Methods of Multiplication, especially with the Roman Notation" by Ruth F. Avery '19.

April 26: "Zeno's Paradoxes" by Ruth F. Avery '19.

May 10: "The Mathematics of Actuarial Work" by Justine McGowan '20.

THE UNIVERSITY OF SASKATCHEWAN MATHEMATICAL SOCIETY, Saskatoon, Saskatchewan.

This Society was formed on November 16, 1916, with an initial membership of 12. The average attendance now is about 14. All students interested in mathematics are eligible for membership.

Officers, 1917–18: Honorary president, Professor L. L. Dines; president, Rhoda S. Russell '19; secretary-treasurer, Nelson W. Taylor '18; program convenor, John H. Simester '20.

The following meetings of the Society have been held:

November 30, 1916: "Flatland" by Nelson W. Taylor '18.

January 19, 1917: "Non-Euclidean Geometry, Lobachevsky's System" by Roy E. Shuttleworth '18.

February 6: "The Squaring of the Circle" by Edward J. Baldes '18; "A Problem on Refraction of Light" by John H. Simester '20.

February 20: "The Duplication of the Cube and the Trisection of an Angle" by Oscar C. Bridgeman '18.

March 15: "A History of Algebra" by J. H. Simester '20.

November 16, 1917: Organization Meeting.

December 7: "The Construction of a Honey Bee's Cell" by J. H. Simester '20; "A, B, and C—The Human Element in Mathematics," from S. Leacock's *Literary Lapses*, read by Nelson W. Taylor '18.

January 28, 1918: "The Fundamental Theorems for Geometric Constructions by Means of the Compasses alone" by Oscar C. Bridgeman '16.

February 15: "The History and Computation of Logarithms" by J. H. Simester '20.

March 1: "Application of the Compasses alone to certain Geometric Constructions including Regular Polygons" by N. W. Taylor '18.

March 15: "Interesting Relations between Plane and Spherical Geometry" by Gladys Shannon '21; "Parallel Postulates and Riemann's System of Geometry" by Frances Schiltz '14.

Pi Mu Epsilon Fraternity, Syracuse University, Syracuse, N. Y.

The Mathematical Club of Syracuse University was founded in November, 1903. After ten years of successful operation it was reorganized into the Pi Mu Epsilon Fraternity, which aims to promote mathematics and scholarship. On May 25, 1914, this fraternity was incorporated under the laws of the State of New York. Among the powers secured in accordance with the articles of incorporation is that of granting charters to other chapters to be organized elsewhere.

Membership in the chapter is open to "members of the mathematical faculty, former members of the club, any person whose work in the mathematical sciences is distinguished, former students in mathematics, and major and minor students who have taken certain specified courses in the subject and who have attained a certain standard of scholarship set by the chapter." There are 44 members in the university this year, 16 of the faculty and graduates, and 28 undergraduates. The average attendance at meetings was 30.

Officers, 1917–18: Director, Professor Warren G. Bullard; vice-director, Professor John L. Jones; secretary, Helen N. Hale '18; treasurer, Howard A. DoBell

'19; librarian,[1] Cornelia A. Tyler '19. Executive committee: the above officers and Christabel A. Christie '18, Edna R. Howe '18, Charles D. Hurd '18, Rennie B. Smith '18. Scholarship committee: Professors Floyd F. Decker and Louis Lindsey, and Helen N. Hale '18, M. Gladys Medbery '18 and Lawrence J. Blackmar '18.[2]

October 23, 1916: Election of new faculty members. Professor L. Lindsey reported concerning the summer meeting of the Mathematical Association of America. Other members reported on their summer work.

November 13: Report of scholarship committee.[3] A committee was appointed to cast the ballot of Pi Mu Epsilon for officers in the Mathematical Association of America.

December 4: Initiation of new members. Committee on new books appointed. "The Development of the Use of Imaginary Numbers" by Beatrice Reynolds '17.

December 18: Report of the committee on new books. Christmas Party.

February 5, 1917: "Integral Equations" by Professor J. L. Jones.

February: Sleighing Party.

February 26: "Aggregates" by Johanna B. Guelzow '17; "The Brachistochrone" by Harold Hendershot '17.

March 19: "The Sphere in Four-Dimensional Space" by Mildred McKay '17.

March 31: Annual banquet.

April 16: "Non-Euclidean Geometry" by Florence Wilcox '17.

May 7: "A Discussion of Some New Curves" by Leon V. Foster '17.

May: Picnic.

October 8, 1917: Election of new faculty members. Reports of summer experiences by Professors Edward D. Roe and Decker.

October 30: "The Emblems of Pi Mu Epsilon" by H. A. DoBell '18; "The History and Ideals of Pi Mu Epsilon" by Professor Roe.

November 19: "Representation of Complex Loci" by Professor Roe.

December 10: "Some Applications of Mathematics to Modern Warfare" by Lawrence J. Blackmar '18.

February 11, 1918: "Applications of Mathematics to Economics" by Professor J. L. Jones; the scholarship grades of the undergraduate members of the chapter for the first semester were read by the scholarship committee and discussed, and the grades were put on record in the minutes.

March 11: "The Relation of Mathematics to Physics and Theoretical Chemistry" by Edna Howe '18; "The Nature of the Atom" by John J. Hopfield '16.

---

[1] A large number of standard mathematical works have been bought by the chapter and are kept in the mathematical seminary. The annual dues are one dollar, yet as much as $30 has been spent for books in a single year.

[2] The students of this committee are those of highest rank in the senior class.

[3] After discussion the minimum scholarship standard for a new student member was set at "cum laude for general work and near to magna cum laude for mathematical work, but in the case of sophomores the latter was put distinctly above magna cum laude."

April 8: "The Impossibility of Squaring the Circle" by M. Gladys Medbery '18; "Geometric Interpretations of Hyperbolic Functions" by Helen H. Hale '18; "Some Ideas of Statistics" by Christabel A. Christie '18; reports of committees on the annual banquet and on new books.

April 29: "Relativity" by Charles D. Hurd '18; "Application of Mathematics to Every Day Life" by Doris A. Bourne '18; Review of Skinner's *Theory of Investments* by Ruth Taylor '18; election of officers for 1918–19.

May 4: Annual banquet.

"It is the duty of every member who presents a paper to copy it in a blank book provided for that purpose. We have three large volumes of such papers in our library—covering the time from 1903 to the present."

THE PENTAGRAM, University of Texas, Austin, Texas.

As the result of Professor Albert A. Bennett's initiative, The Pentagram was organized on October 18, 1916, "to assist in promoting the interests of mathematics among the students of the University of Texas." Papers and problems by both the students and staff of instruction constituted the program of biweekly meetings. "The heaviest duty fell of course on Professor Bennett, who suggested most of the problems as well as the subjects of most of the papers. His unwearying enthusiasm was contagious and the interest never flagged. The club counted over 30 members[1] and at the close of the term a banquet was held." [2]

The name of the club was chosen by the charter members for two reasons: "One was because the pentagram was intimately connected with the beginnings and early growth of mathematics; and the other was because of its significance to every Texan, Texas being called the Lone Star State, there being five letters in the word, Texas having fought under five flags and having served under five governments, and the State seal being a five-pointed star artistically wreathed and engraved."

Under its constitution the club regards as eligible to membership: (1) "those students who have completed an equivalent of five terms of work in mathematics with an average grade (in mathematics) of B; (2) all graduate students in mathematics and persons in the city teaching mathematics of high-school grade; and (3) those students who have completed mathematics 3 (first-year calculus) or its equivalent and are continuing the study of mathematics. Other persons may be elected to membership by a two thirds vote of all members present at a regular meeting and with the approval of the council (which consists of the president, vice-president, secretary-treasurer, student member, and faculty adviser). All members of the mathematical faculty teaching staff are invited to join. Other members are elected after an invitation by the council."

The Club uses printed cards of invitation (3×5 inches) with the following inscription: "The Pentagram invites to Membership . . . . This Club

---

[1] Average attendance about 20.

[2] "Mathematics Clubs," *The Texas Mathematics Teachers' Bulletin*, April 1, 1917, Vol. 2, p. 33. Professor Bennett is now captain in the coast artillery.

meets twice a month to consider mathematical topics of interest to the undergraduate. Dues, one dollar, payable in the winter term. Address acceptances to . . . . Secretary."

A facsimile of The Pentagram's certificate of membership is given on the opposite page. The original is printed on cardboard.

Officers, 1917–18: President, Charles E. Normand '19; vice-president, Ruth Stocking '18; secretary-treasurer, Dr. Goldie P. Horton, instructor in pure mathematics; faculty member of the council, Professor Thomas McN. Simpson; student member of council, Lloyd Kerr '18.

The following programs have been given:

October 18, 1916: Organization meeting; short talk by Professor Milton B. Porter on "What a Mathematics Club can Accomplish."

November 1: "History and Significance of the Name Pentagram," by Clarence E. Brand '17; "The History of Algebraic Notation" by Henry H. Hammer '18.

November 15: "Intuition in Mathematics" by Professor Porter.

December 6: "Some Propositions in Number Theory" by Amelia K. Benson '17.

January 10, 1917: "Map-making" by Rufus R. Rush '16.

January 24: "Trigonometrical Series" by Hyman J. Ettlinger, instructor [at this time] in applied mathematics.

February 7: "The Computation of Logarithm Tables" by Thomas B. McCarter '16.

February 21: "Point Sets" by Dr. Goldie P. Horton.

March 21: Roll-call program, each member responding to roll-call with something of mathematical interest: an incident, quotation, problem, or the like.

April 4: "Some mathematical Instruments: Planimeter, Slide-Rule, Inversors, Sextant" by Llewellyn Notley '17.

April 19: "Some mathematical Problems: The Pythagorean Relation, Some Number Relations, Some Analysis Situs Relations" by Professor A. A. Bennett.

May 3: "Philosophical and Mathematical Views of Infinity" by Clarence E. Brand '16.

October 17, 1917: "Why We Study Mathematics" by Professor M. B. Porter.

October 31: "Newton, Man and Mathematician" by Professor T. McN. Simpson.

November 14: "The Dimensionality of Space as dependent on the Choice of the Elements" by Lloyd Kerr '18.

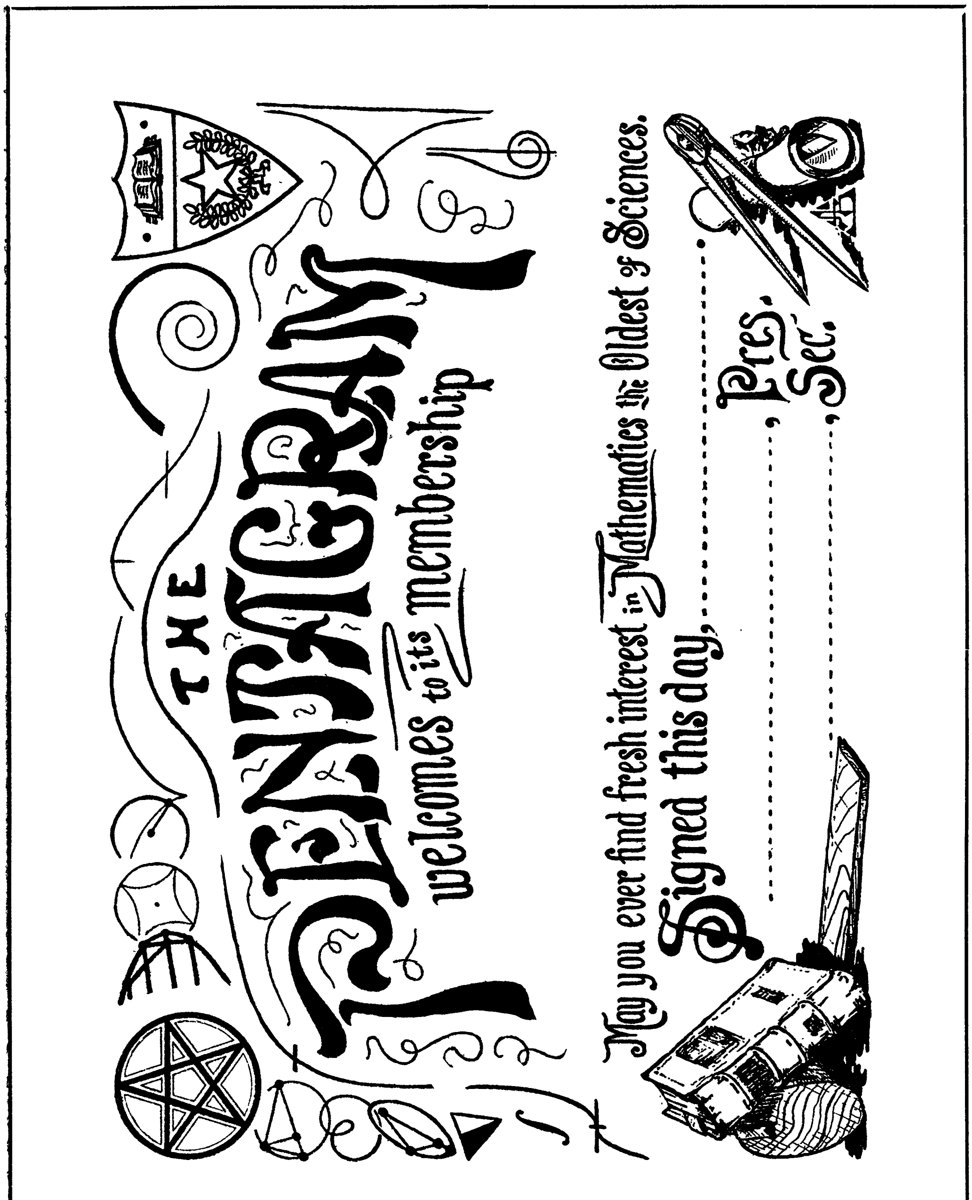
November 28: "The Equation of Life" by Ruth Stocking '18.

December 1: Open meeting, "How we aim the Big Gun"[1] by Professor A. A. Bennett, Captain C. A. R. C.

December 12: "Dialling" by Professor Porter; "The Equation of Time" by Professor Simpson.

January 16, 1918: "The Mathematical Theory of the Chemical Balance" by Charles E. Normand '16.

[1] Cf.. "Remarks on mathematics of artillery," by A. A. Bennett, in *The Texas Mathematics Teachers' Bulletin*, May, 1918, pp. 9–16.


THE
PENTAGRAM
welcomes to its membership
May you ever find fresh interest in Mathematics the Oldest of Sciences.
Signed this day, ..., Pres.
............, Sec.

January 30: "The Theory of Flight" by H. J. Ettlinger, adjunct professor of applied mathematics.

February 13: "Continued Fractions" by Bertha Potash '18.

February 27: "Railway Transition Curves" by Marvin Nichols '18.

March 20: "The Logarithm as a Direct Function" by Dr. Paul M. Batchelder, instructor in applied mathematics.

April 4: "The Use of the Mean-Value Theorem in Finding Linear and Curvilinear Asymptotes" by Robert G. Wulff '20; "The Derivative of Surface Area" by Essie Lipscomb '19.

April 18: "Elementary Properties of Groups" by Lula Whitehouse '19. [Miss Whitehouse was absent on account of illness, and the topic was presented by Dr. Batchelder.]

May 2: "Long Division as Taught in the Middle Ages," by Clara Seymour '18; "The Development of Arabic Numerals" by Phillis Henry '16.

May 16: "Fitting Curves to Biometric Data" by Edward L. Dodd, adjunct professor of actuarial mathematics.

TOPICS FOR CLUB PROGRAMS.

11. Euler Integrals and Euler's Spiral—Sometimes Called Fresnel Integrals and the Clothoïde or Cornu's Spiral.[1]

The integrals in question are

$$\int_0^\infty \sin x^2 dx = \tfrac{1}{2}\int_0^\infty \frac{\sin v}{\sqrt{v}}\,dv$$

and

$$\int_0^\infty \cos x^2 dx = \tfrac{1}{2}\int_0^\infty \frac{\cos v}{\sqrt{v}}\,dv;$$

and the equations of the spiral are

$$x = K\int_0^s \cos s^2 ds, \qquad y = K\int_0^s \sin s^2 ds.$$

These were considered by Euler at least as early as 1743 in a problem of his celebrated work on the calculus of variations: *Methodus inveniendi lineas curvas maxime minimive proprietate gaudentes*[2]. . . . The discussion of the problem was somewhat as follows:[3] Consider an elastic spring freely coiled up in the form

[1] For historical sketches and properties of this curve see F. Gomes Teixeira, *Traité des courbes spéciales remarquables planes et gauches*, tome 2, Coïmbre, 1909, pp. 102–107; G. Loria, *Spezielle algebraische und transzendente ebene Kurven* . . . 2. Auflage. Band 2, Leipzig, Teubner, 1911, pp. 70–73.

[2] Lausannæ & Genevæ, MDCCXLIV, pp. 276–7. *Cf. Verzeichnis der Schriften Leonhard Eulers* bearbeitet von G. Eneström. Erste Lieferung, Leipzig, Teubner, 1910, p. 16. See also P. S. Laplace, "Sur la réduction des fonctions en tables," *Journal de l'École Polytechnique*, tome 8, cahier 15, 1809, pp. 250–251.

[3] As Euler's writings to which reference must be made are very scarce, it would seem best to give more details than otherwise would be necessary.

of a spiral. Let us suppose that the interior extremity is fixed and that the spring can be developed into a horizontal position by a weight p suspended at the other extremity. Under these conditions the action of the weight on an element of the spring placed at a distance s from the extremity is ps; and the elasticity of the element preserves it in equilibrium. This elasticity is "the reciprocal of the osculating radius of the spring in its unrestricted state." Setting r equal to this radius with respect to the part s of the spring, taken from its exterior extremity, we have $ps = Ek^2/r$, Ek^2 being a constant depending upon the elasticity of the spring. Let $Ek^2/p = a^2$. For the spring in its natural position we will thus have (1) $rs = a^2$, "quæ est æquatio naturam curvæ . . . complectens." Whence, introducing rectangular coördinates,

$$\text{“}x = \int ds \sin \cdot \frac{ss}{2aa}, \qquad \& \qquad y = \int ds \cos \cdot \frac{ss}{2aa}.\text{”}$$

Euler then remarks: "From the fact that the osculating radius steadily decreases the longer the arc taken it is evident that the curve is not produced to infinity. The curve therefore will be in the nature of a spiral so that when the spiral is completed it is rolled up, as it were, in a certain point which may be called the center. The point seems to be very difficult to discover by this construction. Therefore we must admit that analysis will make no small gain should anyone find a method whereby, approximately at least, the value of this integral would be determined in the case that s is taken to be infinite. This problem does not seem to be unworthy the best strength of geometers."

Euler then expresses x and y as converging series in s for approximating to these values but adds that if s be made infinite the values of x and y cannot be determined in this way. He sets, finally, $s^2/2a^2 = v$, obtaining

$$(2) \qquad x = \frac{a}{\sqrt{2}} \int \frac{\sin vdv}{\sqrt{v}}, \qquad y = \frac{a}{\sqrt{2}} \int \frac{\cos vdv}{\sqrt{v}},$$

and shows that approximate values of x and y could be found by considering the intervals $v = 0$ to $v = \pi$, $v = \pi$ to $v = 2\pi$, $v = 2\pi$ to $v = 3\pi$, $\cdots$, and certain converging series "requiring long operations and very tedious calculations to evaluate them."

Thirty-eight years later, however, Euler had solved the problem completely. This solution is to be found in one of the last papers which he wrote (he died in 1785). It is entitled "De valoribus integralium a terminus variabilis $x = 0$ usque ad $x = \infty$ extensorum" and was presented to the Academy at Petrograd on April 30, 1781.[1] Again he considers the curve the radius of curvature at each point of which is inversely proportional to the arc of the curve and is led, as before, to the equation $rs = a^2$ from which, Euler says, it would not be difficult to discuss the form of such a curve. He refers to the infinite number of whorls

[1] *Leonhardi Euleri Institutionum calculi integralis*, Vol. 4, Petropoli, 1794, pp. 337–345; editio tertia, Petropoli, 1845, pp. 337–345. German edition: *Vollständige Anleitung zur Integralrechnung* . . . übersetzt von J. Salomon, Wien, 1830, pp. 321–328.

(*infinitas spiras*) about a fixed point "which may be called the pole of this curve." He then proceeds to determine the coördinates of this pole. Introducing the angle of contingence (v) he is led to the form (3)[1] $s^2 = 2a^2v$, from which he finds readily the equations (2). Concerning the evaluation of these integrals for the coördinates of the pole he remarks that he had "recently found by a happy chance and in an exceedingly peculiar manner" that

$$x = \frac{a}{\sqrt{2}} \frac{\sqrt{\pi}}{2}, \qquad y = \frac{a}{\sqrt{2}} \frac{\sqrt{\pi}}{2}.$$

Euler's method of evaluation is based upon that of $\int_0^\infty x^{n-1}e^{-x}dx = \Gamma(n)$ which, in turn, leads to

$$\int_0^\infty x^{n-1}e^{-px}\cdot\cos qxdx = \frac{\Gamma(n)\cos n\alpha}{r^n}, \qquad \int_0^\infty x^{n-1}e^{-px}\sin qxdx = \frac{\Gamma(n)\sin n\alpha}{r^n},$$ [2]

where $p = r\cos\alpha$, $q = r\sin\alpha$. Euler then sets $q = 1$, $p = 0$, $n = \frac{1}{2}$ and finds the required result. He notes also:

$$\int_0^\infty \frac{e^{-px}\cos qxdx}{\sqrt{x}} = \sqrt{\frac{\pi}{r}}\cos\frac{\alpha}{2},$$ [3] $$\int_0^\infty e^{-px}\sin\frac{qx}{\sqrt{x}}dx = \sqrt{\frac{\pi}{r}}\sin\frac{\alpha}{2},$$

[1] A curve with an equation of this same form was named by K. C. F. Krause "parabola originaris longitudinaris" (*Nova theoria linearum curvarum*, Monachii, anno MDCCCXXXV, p. 79). The reason for the name is clear. None of Krause's discussion of the curve is worth referring to; Loria's mention of it seems misleading in part.

[2] These were the integrals investigated by Poisson in his "Mémoire sur les intégrales définies," *Journal de l'École Polytechnique*, Paris, tome 9, cahier 16, pp. 215–246, 1813, especially pp. 215–219. See also Poisson, *Nouveau Bulletin des Sciences* par la société philomathique de Paris, 3 année, 1811, tome 2, p. 251; Lacroix, *Traité du calc. diff. et integ.*, tome 3, 2e éd., Paris, 1819, pp. 486–490; Grunert, Crelle's *Journal*, Band 8, 1832, pp. 146–151; J. Plana, "Sur trois intégrales définies," *Acad. Sci. Mém.*, Bruxelles, Vol. 10, 1837; A. De Morgan, *Differential and Integral Calculus*, London, 1842, p. 630; Schlömilch, "Ueber einige Integrale welche goniometrische Funktionen involvieren," *Arch. d. Math. u. Phys.*, 1845, Band 6, pp. 200 ff.; E. F. A. Minding, "Ueber $\int_0^\infty \sin x^m \cdot x^{-n}dx$ wo $m \geqq n > 0$," *Arch. d. Math. u. Phys.*, Band 30, 1858, pp. 171–183; W. Walton, "On a Pair of Definite Integrals," *Quarterly Journal of Mathematics*, 1871, Vol. 11, pp. 373–375; J. W. L. Glaisher, "On certain Definite Integrals," *Report of the British Association for the Advancement of Science*, 1871, London, 1872, Report, pp. 10–12.

[3] For $p = 1$ and $q = 0$ we find, on substituting t^2 for x, that $\int_0^\infty e^{-t^2}dt = \sqrt{\pi}/2$, an integral (of great importance in many parts of applied mathematics) definitely evaluated by Laplace in a memoir published in 1781 (*Mém. Acad. Paris*, 1778; *Œuvres*, Paris, Vol. 9, "Mémoire sur les probabilités"). The Euler integrals, and spiral in connection with the elastic spring, of these notes were also discussed by Laplace in "Sur la réduction des fonctions en tables," *Journal de l'École Polytechnique*, tome 8, cahier 15, pp. 229–265, 1809. A slip made by Mascheroni is here corrected; in his *Adnotationes ad calculum integralen Euleri* ec. (Ticini, M.DCCXC; [also L. Euler, *Opera Omnia*, series 1, Vol. 12, Leipzig, Teubner, 1914]) Mascheroni has a note on Cap. V, Sect. I, Vol. 1, entitled: "De integratione Formularum $x^ndx \sin x$, $x^ndx \cos x$," pp. 38–57 [pp. 454–471]. In the special case $n = -\frac{1}{2}$ he gives (p. 53) $\sqrt{2\pi}$, instead of $\sqrt{\pi/2}$.

and the well-known result

$$\int_0^\infty \frac{\sin x}{x}\,dx = \frac{\pi}{2},\text{[1]}$$

the evaluation of which "up to the present has defeated all known artifices of calculation."

While Euler was not the first to discuss some of the problems mentioned above, he was the first to publish any results of importance in connection with them. In 1694 (at the close of his memoir "Curvata Laminæ Elasticæ"[2]) James Bernoulli (1654–1705) mentions among problems which might be worked out: To find the curvature a lamina should have in order to be straightened out horizontally by a weight suspended at one end. In the posthumous edition of his works, in 1744, there is a reference from this passage to a fragment entitled "Invenire Curvarum, cujus curvedo in singulis punctis est proportionalis longitudini arcus; id est, quæ ab appenso pondere flectitur in rectam."[3] Here there is the equation $rs = a^2$ and a construction for points on the curve; but there is not the slightest indication that Bernoulli had any conception, such as Euler had, of the real form of the curve.

In the nineteenth century the Euler integrals and spiral became of special interest through discoveries of Fresnel (1788–1827) in connection with the diffraction of light. By making certain assumptions and approximations Fresnel deduced (in 1818)[4] for the intensity of the illumination at any point of a diffraction pattern

$$I = [\textstyle\int \cos \frac{1}{2}\pi v^2 dv]^2 + [\textstyle\int \sin \frac{1}{2}\pi v^2 dv]^2.$$

For this reason the integrals which here occur are often called Fresnel's integrals. From what has been indicated above the value of each, for the limits $v = 0$ to $v = \infty$, is $\frac{1}{2}$.

In his Note of 1818, Fresnel gave a table of the values of $A = \int_0^v \cos \frac{1}{2}\pi v^2 dv$ and $B = \int_0^v \sin \frac{1}{2}\pi v^2 dv$ for values of v (differing by 0.1) from 0.1 to 5.1 (later extended to 5.5), to 4 places of decimals. This table is reproduced by E. Verdet in his *Leçons d'Optique physique* (1869).[5] More detailed tables (to the nearest hundredth) were calculated by Abria.[6] Modifications of Fresnel's method of evaluating A and B, and criticisms and corrections of his results were given by

[1] See G. H. Hardy's discussion of eleven proofs of this in *Mathematical Gazette*, July, 1916, Vol. 8, pp. 301–303; see also Vol. 5, pp. 98–103, 1909, and Vol. 6, pp. 223–4, 1912.

[2] *Acta Eruditorum*, 1694, p. 276; *Opera*, Genevæ, M.DCCXLIV, tome 1, p. 600.

[3] J. Bernoulli, *Opera*, tome 2, pp. 1084–1086.

[4] *Œuvres complètes d'Augustin Fresnel*, tome 1, Paris, MDCCCLXVI, pp. 176–181; see also pp. 198–9, 315–352.

[5] Publiées par A. Levistal, tome 1, Paris, 1869; *cf.* pp. 343 ff. German edition by K. Exner, Band 1, Braunschweig, Vieweg, 1881, pp. 236–309; the table is given on p. 240 and on p. 241 A and B are graphed as oscillating curves.

[6] "Sur la diffraction de la lumière," *Journal de mathématique pures et appliquées*, 1839, tome 4, pp. 248–260.

Knochenhauer,[1] Cauchy,[2] and Gilbert.[3] Knochenhauer's method was good for small values and Cauchy's for large values of v. Peters gives (*l. c.*, page 48) a table similar to that of Fresnel but slightly more extensive, in that intervals of 0.01 are considered from $v = 0.01$ to $v = 0.10$, and intervals of 0.05 from $v = 0.10$ to $v = 1.00$. This is the table most frequently quoted;[4] but it should be remembered that, except for some corrections and slight additions, it is identical with Fresnel's given some forty years earlier.

The tables of Ignatowsky[5] give (among other things), from $v = 0.0$ to $v = 8.5$ (for intervals 0.1), the values of A and B to four places of decimals, and of $\log A$ and $\log B$ to six places. Lommel published[6] a table for

$$A' = \int_0^z \frac{\cos z}{\sqrt{z}}\, dz, \qquad B' = \int_0^z \frac{\sin z}{\sqrt{z}}\, dz$$

from $z = 0$ to $z = 50$ at unit intervals. From $z = 0.0$ to $z = 50.0$ at intervals of 0.1, and to four places, it is printed in Jahnke and Emde's tables.[4]

In 1874 Cornu plotted Euler's spiral accurately[7] by means of Peters's table. (Euler had already given half the spiral.) In a sketch of Cornu, Poincaré has written as follows:[8] "Aussi, quand il aborda l'étude de la diffraction, il eut bientôt fait de remplacer cette multitude rebarbative de formule herissées d'intégrales par une figure unique et harmonieuse, que l'oeil suit avec plaisir et où l'esprit se dirige sans effort. Tout le monde aujourd'hui pour prévoir l'effet d'un écrau quelconque sur un faisceau lumineux, se sert de la *spirale de Cornu.*" The expression "Cornu's spiral" was used by Preston,[9] Wood,[10] and others before Poincaré[11] employed it in the sketch quoted; but the term is evidently highly inappropriate in the light of Euler's discoveries set forth above.

Besides the works on physics to which reference has been made already in

[1] K. W. Knochenhauer (1) "Ueber die Oerter der Maxima und Minima des gebeugten Lichtes nach den Fresnel'schen Beobachtungen," *Annalen der Physik und Chemie*, Leipzig, 1837, Band 41, pp. 103–110; (2) "Ueber eine besondere Klasse von Beugungserscheinungen," *idem*, 1838, Band 43, pp. 286–292; (3) *Die Undulationstheorie des Lichtes*, Berlin, 1839, p. 36f.

[2] A. Cauchy, *Comptes Rendus*, Paris, 1842, tome 15, "Note sur la diffraction de la lumière," pp. 554–6; "Addition à la Note sur la diffraction de la lumière," pp. 573–578.

[3] Ph. Gilbert, "Recherches analytiques sur la diffraction de la lumière" (mémoire présenté le 3 août, 1861). *Mémoires couronnés . . . acad. roy. d. sc. . . . de Belgique*, 1863, tome 31, pp. 1–52.

[4] For example: E. Jahnke und F. Emde, *Funktionentafeln mit Formeln und Kurven*, Leipzig, 1909, pp. 23–26; R. W. Wood, *Physical Optics*, New York, Macmillan, 1905, p. 198.

[5] W. v. Ignatowsky, *Annalen der Physik*, 1907, Band 328, pp. 895–898.

[6] E. Lommel, *Abh. Münch. Ak.*, Band 15, 2. Abtheilung, 1880, p. 230.

[7] A. Cornu, (1) "Méthode nouvelle pour la discussion des problèmes de diffraction dans le cas d'une onde cylindrique," *Journal de physique théorique et appliquée*, Paris, 1874, pp. 5–15; (2) "Études sur la diffraction; méthode géométrique pour la discussion des problèmes de diffraction," *Comptes Rendus*, tome 78, 1874, pp. 113–117.

[8] H. Poincaré, *Savants et écrivains*, Paris, Flammarion, [1910], p. 106.

[9] T. Preston, *The Theory of Light*, 2d edition, London, Macmillan, 1895, p. 274 [4th ed., 1912, p. 291].

[10] R. W. Wood, *Physical Optics*, New York, Macmillan, 1905, p. 158 and on the plate at the end of the volume.

[11] Loria seems to be in error here (*l. c.*, p. 71).

connection with our topic we may note those by Drude,[1] Pockels,[2] and Chwolson.[3]

Lommel seems to have been the first[4] to observe the connection between A', B' and Bessel's functions:

$$\int \frac{\cos z}{\sqrt{z}} dz = \sqrt{\frac{\pi}{2}} \int J_{-1/2}(z) dz; \qquad \int \frac{\sin z}{\sqrt{z}} dz = \sqrt{\frac{\pi}{2}} \int J_{1/2}(z) dz.$$

Amongst the many methods for evaluating A and B reference may be given: (1) to the method of Godefroy[5] who, by a slight modification of Laurent's discussion,[6] starts with $\int_0^\infty e^{-x^2}dx = \sqrt{\pi}/2$ and avoids all use of imaginaries; (2) to a paper by Cayley[7] discussing several interesting points which later occupied the attention of Glaisher,[8] Jamet,[9] and Humbert;[10] (3) to other methods illustrated by Pierpont[11] and d'Adhémar;[12] and (4) to Noumoff's "Interprétation géométriques des intégrales de Fresnel" [13] in which the projection of helices generated by a certain parabola rolling on a right circular cylinder are curves the sum of the areas under which give the required values. The latter part of the article contains a description of a mechanical integrator for calculating the integrals A and B.

In recent times Cesàro has given to Euler's Spiral the name Clothoïde[14] and exhibited a number of remarkable properties of the curve.[15] Among these the

[1] P. Drude, *Lehrbuch der Optik*, Leipzig, Hirzel, 1900, pp. 174–187; English translation by C. R. Mann and R. A. Millikan, London, Longmans, 1902, pp. 188–202.

[2] F. Pockels, pp. 1051–1064 of *Handbuch der Physik*, Zweite Auflage herausgegeben von A. Winkelmann, Band 6: Optik, Leipzig, Barth, 1906.

[3] O. D. Chowlson, *Traité de physique*, ouvrage traduit sur les éditions russe et allemande, tome 2, Paris, Hermann, 1909, pp. 652–656.

[4] E. Lommel, "Ueber die Anwendung der Bessel'schen Funktionen in der Theorie der Beugung," *Zeitschrift für Mathematik und Physik*, Leipzig, 1870, pp. 141–169. *Cf.* A. Gray and G. B. Mathews, *Treatise on Bessel Functions*, London, 1895, p. 41.

[5] A. Godefroy, "Sur les intégrales de Fresnel," *Nouvelles annales de mathématiques*, 1898 (3), Vol. 17, pp. 205–206.

[6] H. Laurent, *Traité d'analyse*, tome 3, Paris, 1888, p. 137.

[7] A. Cayley, "Note on the Integrals $\int_0^\infty \cos x^2\, dx$ and $\int_0^\infty \sin x^2\, dx$," *Quarterly Journal of Mathematics*, Vol. 12, 1873, pp. 118–126; also *Collected Mathematical Papers*, Cambridge, Vol. 9, 1896, pp. 56–63.

[8] J. W. L. Glaisher, "On the Integrals $\int_0^\infty \sin x^2 dx$ and $\int_0^\infty \cos x^2 dx$, *Quarterly Journal*, 1875, Vol. 13, pp. 343–349.

[9] V. Jamet, "Sur les Intégrales de Fresnel," *Nouvelles annales de mathématiques*, 1896 (3), tome 15, pp. 372–376.

[10] G. Humbert, *Cours d'analyse*, Paris, Gauthier-Villars, tome 1, 1903, pp. 307–08.

[11] J. Pierpont, *Lectures on the Theory of Functions of Real Variables*, Boston, Ginn, Vol. 1, 1905, pp. 499–500.

[12] R. d'Adhémar, *Exercises et Leçons d'Analyse*, Paris, Gauthier-Villars, 1908, pp. 23–25.

[13] *Journal de physique théorique et appliquée*, Paris, 1847 (3), tome 6, pp. 281–289.

[14] From the Greek word meaning to twist by spinning—since the curve spins or turns about its asymptotic points.

[15] E. Cesàro, (1) "Les lignes barycentriques," *Nouvelles annales de mathématiques*, 1886 (3), tome 5, pp. 511–520; (2) "Sur la courbe représentative des phenomènes de diffraction," *Comptes Rendus*, 1890, tome 110, pp. 1119–1122; (3) *Nouvelles annales de mathématiques*, 1905, 4e série, tome 5, pp. 570–573. See also *L'Intermédiaire des mathématiciens*, 1916, tome 23, pp. 187–189.

following may be mentioned: (*a*) The clothoïde is the only curve enjoying the property that the center of gravity of any arc is a center of similitude of the circles osculating the extremities of the arc; (*b*) when a clothoïde rolls on a straight line, the locus of the center of curvature corresponding to the point of contact is an equilateral hyperbola asymptotic to the line considered.

Wieleitner discussed "Die Parallelkurve der Klothoide."[1] For different values of m the intrinsic equation $rs^m = a^2$ represents a clothoïde, a logarithmic spiral, a circle, the involute of a circle, and a straight line.

NOTES AND NEWS.

EDITED BY D. A. ROTHROCK, Indiana University, Bloomington, Indiana.

Assistant Professor J. L. COOLIDGE, of Harvard University, has been promoted to an associate professorship of mathematics.

Dr. ANNA J. PELL, of Mount Holyoke College, will fill the vacancy at Bryn Mawr College caused by the resignation of Dr. OLIVE C. HAZLETT.

At Leland Stanford Junior University Dr. H. C. MORENO has been promoted from an assistant to an associate professorship of applied mathematics.

Dr. FLORA E. LE STOURGEON, of the Liggett School, Detroit, Mich., has accepted an instructorship in mathematics at Mount Holyoke College.

At the University of Illinois Mr. R. F. BORDEN, a graduate student, has been appointed instructor in mathematics, and Mr. JOSEPH ROSENBACH, of the University of New Mexico, has been appointed assistant in mathematics.

Professor R. E. MORITZ, of the University of Washington, has in *School and Society*, April 27, 1918, a Reply to ERNEST C. MOORE's paper "Does the study of mathematics train the mind specifically or universally?", which was published in the same periodical on October 27, 1917.

A fund of 150,000 crowns has been donated by Mr. C. HENNEVIG as a memorial to N. H. ABEL, the income from which is to be used to encourage mathematical research in Norway.

Professor E. V. HUNTINGTON, President of the Mathematical Association of America, has taken leave of absence from Harvard University and with the rank of major in the national army is assigned to statistical study under the chief of staff with residence in Washington.

Professor W. D. CAIRNS, of Oberlin College, Secretary of the Mathematical Association of America, is giving courses in the fundamental concepts of algebra and geometry and in the teaching of mathematics, and a graduate course in the calculus of variations in the summer session of Ohio State University.

---

[1] *Archiv der Mathematik und Physik*, 1907 (3) Band 11, pp. 373–375.